INUKSHUK ACADEMY

Course code: **Advanced Functions (MHF4U)**
Total Hours: **110 hours** and Credit: **1.0**
Prerequisite: **MPM2D, Grade 10 Principles of Mathematics**
Curriculum basis: **Ontario Curriculum, Grades 11–12: Science (Revised 2008)**
Last Update: **Inukshuk Academy 2026**

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Course Description

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

1. Course Rationale

Advanced Functions provides the theoretical and mathematical framework for understanding complex relationships, patterns, and change. MHF4U strengthens students' ability to analyze advanced function models, interpret mathematical behaviour, and construct logical arguments using algebraic, graphical, numerical, and verbal representations.

Through inquiry, problem solving, and modelling, students:

- Develop a deeper understanding of polynomial, rational, exponential, logarithmic, trigonometric, and composite functions;
- Explore how functions can be transformed, combined, compared, and applied to real-world contexts;
- Refine algebraic, graphical, and analytical skills essential for Calculus and Vectors and university-level mathematics;
- Learn to analyze rates of change, solve multi-step problems, and communicate mathematical reasoning clearly;
- Recognize the role of advanced functions in science, engineering, business, technology, and everyday decision-making.

The course fosters mathematical literacy, critical thinking, and problem-solving fluency, skills essential for success in university-level programs and careers in mathematics, science, engineering, business, economics, computer science, and technology.

2. Overall Curriculum Expectations

The Ontario Curriculum organizes MHF4U into four major strands. The following expectations are paraphrased for clarity and alignment with Inukshuk Academy's course design.

a. Exponential and Logarithmic Functions

Students will:

- Demonstrate an understanding of the relationship between exponential and logarithmic expressions;

- Analyze and graph exponential and logarithmic functions using multiple representations;
- Apply exponent and logarithm laws to simplify expressions and solve equations;
- Solve real-world problems involving growth, decay, finance, measurement scales, and scientific models.

b. Trigonometric Functions

Students will:

- Use radian measure to describe angles, rotations, and periodic behaviour;
- Analyze and graph trigonometric and reciprocal trigonometric functions;
- Identify amplitude, period, phase shift, vertical shift, asymptotes, and key features of trigonometric graphs;
- Prove trigonometric identities and solve trigonometric equations over specified intervals;
- Model and solve real-world problems involving periodic relationships.

c. Polynomial and Rational Functions

Students will:

- Investigate the structure and behaviour of polynomial functions;
- Analyze zeros, turning points, end behaviour, intervals of increase and decrease, and transformations;
- Interpret and graph rational functions, including restrictions, asymptotes, holes, and discontinuities;
- Solve polynomial and rational equations and inequalities using algebraic and graphical methods.

d. Characteristics of Functions

Students will:

- Investigate average and instantaneous rates of change using numerical, graphical, and algebraic methods;
- Combine functions using addition, subtraction, multiplication, division, and composition;
- Analyze the domain, range, and behaviour of combined and composite functions;
- Compare characteristics of different function types and justify their use in mathematical modelling.

3. Learning Goals for Students

By the end of MHF4U, students will be able to:

- Analyze polynomial, rational, exponential, logarithmic, trigonometric, and composite functions using algebraic, graphical, numerical, and verbal representations;

- Apply transformations to advanced functions and interpret their effects on graphs, equations, domain, range, intercepts, asymptotes, and end behaviour;
- Solve problems involving polynomial and rational equations and inequalities, trigonometric equations and identities, and exponential and logarithmic models;
- Combine functions using algebraic operations and composition, and analyze the resulting function behaviour;
- Investigate average and instantaneous rates of change using numerical, graphical, and algebraic methods;
- Model real-world relationships involving growth, decay, periodic behaviour, rates of change, and complex functional relationships;
- Communicate mathematical reasoning clearly using appropriate notation, terminology, graphs, technology, and justified solution steps.

Course Structure and Instructional Units

Unit	Description	Hours
Unit 1: Polynomial and Rational Functions	Polynomial behaviour, zeros, turning points, end behaviour, factor and remainder theorems, rational functions, asymptotes, restrictions, equations and inequalities	26
Unit 2: Trigonometric Functions	Radian measure, sinusoidal and reciprocal functions, transformations, identities, compound-angle formulas, trigonometric equations	25
Unit 3: Exponential and Logarithmic Functions	Exponent and logarithm laws, transformations, exponential and logarithmic equations, growth and decay models	24
Unit 4: Characteristics of Functions	Function operations, composition, domain and range, average and instantaneous rates of change, modelling and comparison of functions	22
Culminating Task	Mathematical investigation or applied modelling project	8
Final Exam	Proctored examination	5
	Total	**110**

4. Course Structure, Unit Overview and Instructional Plan

1. Unit 1: Polynomial and Rational Functions

Approx. Hours: 26

a. Unit Overview

In this unit, students investigate the properties and behaviour of polynomial and rational functions using algebraic, graphical, numerical, and analytical methods. Students analyze polynomial functions by examining degree, zeros, multiplicity, end behaviour, turning points, and transformations. They apply factoring techniques, the factor theorem, and the remainder theorem to solve equations and model mathematical relationships.

Students also explore rational functions, including restrictions, asymptotes, discontinuities, and reciprocal relationships. Technology is used extensively to visualize function behaviour and verify algebraic reasoning.

b. Key Learning Goals

Students will:

- Analyze polynomial functions and identify key graphical characteristics;
- Determine zeros, multiplicity, intercepts, and end behaviour of polynomial functions;
- Apply factoring techniques, the factor theorem, and the remainder theorem;
- Analyze rational functions, including asymptotes and discontinuities;
- Solve polynomial and rational equations and inequalities using algebraic and graphical methods.

c. Sample Learning and Assessment Activities

- Polynomial graphing investigations using technology;
- Factor and remainder theorem explorations;
- Rational function asymptote analysis tasks;
- Real-world modelling problems involving polynomial functions;
- Graphing assignments, algebraic investigations, quizzes, unit test.

2. Unit 2: Trigonometric Functions

Approx. Hours: 25

a. Unit Overview

Students extend their understanding of trigonometry through the study of radian measure, trigonometric and reciprocal trigonometric functions, transformations, identities, and equations. Students investigate periodic behaviour using graphical and algebraic representations while analyzing amplitude, period, phase shift, vertical translation, and asymptotic behaviour.

Students also prove identities, solve trigonometric equations over specified intervals, and apply trigonometric models to real-world contexts involving motion, sound, waves, and cyclical behaviour.

b. Key Learning Goals

Students will:

- Convert between degree and radian measure;
- Analyze and graph trigonometric and reciprocal trigonometric functions;
- Apply transformations to trigonometric functions;
- Prove and apply trigonometric identities;

- Solve trigonometric equations algebraically and graphically;
- Model periodic relationships using sinusoidal functions.

c. Sample Learning and Assessment Activities

- Unit circle and radian investigations;
- Reciprocal function graphing activities;
- Trigonometric identity proofs and equation-solving tasks;
- Sinusoidal modelling investigations using technology;
- Trigonometry assignments, graphing investigations, quizzes, unit test.

3. Unit 3: Exponential and Logarithmic Functions

Approx. Hours: 24

a. Unit Overview

In this unit, students investigate exponential and logarithmic functions and the inverse relationship between them. Students simplify expressions using exponent and logarithm laws, graph exponential and logarithmic functions, and solve equations involving exponents and logarithms.

Applications involving population growth, compound interest, radioactive decay, pH scales, sound intensity, and scientific measurement provide meaningful real-world contexts for mathematical modelling and interpretation.

b. Key Learning Goals

Students will:

- Apply exponent and logarithm laws to simplify expressions;
- Analyze and graph exponential and logarithmic functions;
- Solve exponential and logarithmic equations;
- Interpret growth and decay models in applied contexts;
- Compare logarithmic and exponential relationships using multiple representations.

c. Sample Learning and Assessment Activities

- Exponential growth and decay modelling tasks;
- Logarithmic equation-solving investigations;
- Graphing explorations using technology;
- Financial and scientific application case studies;
- Modelling assignments, quizzes, equation-solving tasks, unit test.

4. Unit 4: Characteristics of Functions

Approx. Hours: 22

a. Unit Overview

Students investigate advanced characteristics of functions, including operations on functions, composition of functions, inverse functions, and rates of change. Students analyze average and instantaneous rates of change using numerical, graphical, and algebraic methods while comparing behaviours across multiple function types.

The unit emphasizes mathematical reasoning, modelling, and communication through the analysis of complex functional relationships and real-world applications.

b. Key Learning Goals

Students will:

- Combine functions using algebraic operations and composition;
- Determine and analyze inverse functions;
- Investigate average and instantaneous rates of change;
- Compare characteristics of multiple function types;
- Apply advanced function concepts to modelling and problem solving.

c. Sample Learning and Assessment Activities

- Function composition investigations;
- Rate-of-change analysis using graphs and tables;
- Inverse function explorations;
- Technology-based modelling and comparison tasks;
- Investigations, modelling assignments, quizzes, unit test.

Teaching and Learning Strategies

Instruction in MHF4U is grounded in the seven mathematical processes identified in the Ontario Curriculum. These processes guide lesson design, assessment, and classroom interaction to ensure students develop deep conceptual understanding alongside procedural fluency. Teaching strategies emphasize inquiry, exploration, and the thoughtful application of advanced mathematical ideas to real-world and theoretical contexts.

• **Problem solving:** Students engage in rich tasks that require them to apply prior knowledge, test strategies, and make decisions as they work toward solutions. Problems are designed to encourage perseverance, flexible thinking, and advanced mathematical reasoning.

• **Reasoning and proving:** Learners justify steps, explain relationships, and construct logical arguments. Opportunities to prove trigonometric identities, verify algebraic results, analyze rates of change, and examine function behaviour strengthen mathematical reasoning.

• **Reflecting:** Students examine their own thinking, assess the effectiveness of strategies, and revise approaches based on feedback. Reflection is embedded through conferencing, post-assessment reviews, self-assessment, and correction analysis.

• **Selecting tools and strategies:** Learners choose appropriate tools, including graphing technology, algebraic methods, numerical techniques, and modelling strategies, to investigate advanced functions and solve problems. Instruction supports strategic tool use rather than rote application.

• **Connecting:** Students link new concepts to prior learning, relate algebraic and graphical representations, and recognize similarities across polynomial, rational, trigonometric, exponential, logarithmic, and composite functions. Connections to real-world applications help solidify understanding.

• **Representing:** Learners express mathematical ideas using equations, graphs, tables, diagrams, transformations, function notation, and verbal explanations. Multiple representations allow students to approach complex problems from different perspectives.

• **Communicating:** Students articulate reasoning clearly in written, verbal, visual, and symbolic forms. Communication is fostered through discussion, collaborative work, structured mathematical explanations, and justified solution steps.

These strategies collectively ensure that students develop robust conceptual understanding, confidence in problem solving, and the ability to use advanced functions effectively in both academic and applied contexts.

Culminating Task

Approx. Hours: 8

Students complete a mathematical investigation or applied modelling project integrating multiple strands of the course. The culminating task emphasizes advanced problem solving, mathematical communication, reasoning, modelling, and application of advanced functions to real-world contexts.

Final Exam

Approx. Hours: 5

Students complete a proctored final examination evaluating overall expectations from all course strands, including algebraic reasoning, graph interpretation, function analysis, modelling, and problem solving.

Instructional Approaches

Teaching and learning in MHF4U at Inukshuk Academy a wide range of strategies to ensure students develop strong conceptual understanding, analytical thinking, and confidence in applying advanced functions to real-world and theoretical situations. Instruction is guided by the mathematical processes and inquiry-based principles outlined in the Ontario Curriculum.

a. Inquiry-Based and Experiential Learning

Students learn mathematics most effectively through exploration, investigation, and problem solving. Lessons regularly include modelling activities, guided inquiry tasks, technology-based investigations, and real-world applications that reinforce advanced function concepts.

Examples include:

- Investigating polynomial and rational function behaviour using graphing technology;
- Exploring asymptotes, zeros, turning points, and end behaviour through graphical and algebraic analysis;
- Analyzing exponential and logarithmic models in financial, scientific, and environmental contexts;
- Modelling periodic behaviour using trigonometric functions and sinusoidal graphs;
- Investigating average and instantaneous rates of change using tables, graphs, and equations.

b. Multiple Representations of Concepts

Students encounter mathematical ideas through a combination of:

- Algebraic expressions, equations, and function notation;
- Graphs, tables, and diagrams;
- Numerical patterns and data analysis;
- Verbal explanations and mathematical reasoning;
- Technology-based visualizations and dynamic models.

This approach supports deeper conceptual understanding and helps students make connections between different representations of advanced functions and mathematical relationships.

c. Use of Technology and Digital Tools

Digital tools are integrated throughout the course to enhance learning, support inquiry, and strengthen visualization skills. Technology is used to:

- Graph and analyze polynomial, rational, exponential, logarithmic, and trigonometric functions dynamically;
- Model transformations, asymptotes, and rates of change;
- Investigate function operations and composition;
- Compare multiple representations of functions;
- Analyze real-world data and mathematical models;
- Verify algebraic solutions and explore patterns.

Examples include Desmos, GeoGebra, spreadsheets, graphing calculators, and online mathematics simulations.

d. Collaborative Problem Solving

Mathematical learning is strengthened through collaboration. Students work in pairs or small groups to:

- Solve multi-step and non-routine problems;
- Compare and justify solution strategies;
- Investigate mathematical patterns and relationships;
- Communicate and critique mathematical reasoning;
- Complete inquiry and modelling activities collaboratively.

Collaborative learning supports communication skills, critical thinking, and confidence in advanced mathematical problem solving.

e. Differentiated Instruction

Instruction is adapted to support a variety of learning styles, readiness levels, and interests. Strategies include:

- Scaffolded problem sets with increasing complexity;
- Enrichment and extension tasks for advanced learners;
- Visual, numerical, graphical, and algebraic representations to support conceptual understanding;
- Guided practice and targeted intervention for students requiring additional support.

f. Explicit Teaching of Mathematical Processes

Students receive direct instruction and ongoing support in the mathematical processes emphasized in the Ontario Curriculum, including:

- Problem solving;
- Reasoning and proving;
- Reflecting on mathematical thinking;
- Selecting tools and computational strategies;
- Connecting mathematical concepts;
- Representing ideas in multiple ways;
- Communicating mathematical reasoning clearly and accurately.

Assessment and Evaluation

Assessment in MHF4U aligns with the principles of *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools* (2010) [1]. Evaluation reflects students' most consistent achievement, with particular emphasis on recent evidence of learning.

Assessment serves three purposes:

a. Assessment for Learning (Diagnostic)

- Initial review of prerequisite algebra, functions, graphing, trigonometry, and equation-solving skills;
- Diagnostic quizzes to identify prerequisite gaps;
- Informal observations and class discussions;
- Technology-based readiness checks and graphing activities.

b. Assessment as Learning (Formative)

- Homework and practice problem sets;
- Graphing, modelling, and function investigations with iterative feedback;
- Reflection tasks and self-assessment activities;
- Collaborative inquiry and problem-solving exercises;
- Draft submissions of mathematical investigations.

c. Assessment of Learning (Summative)

- Unit tests on major strands;
- Mathematical investigations and modelling assignments;
- Performance tasks involving real-world and theoretical applications;
- Culminating mathematical project;
- Final examination.

d. Achievement Chart Categories

Student achievement is evaluated across the four categories defined in the Ontario Curriculum for Mathematics:

Knowledge & Understanding (30 percent):
Understanding of mathematical concepts, procedures, properties, and relationships involving polynomial, rational, exponential, logarithmic, trigonometric, and composite functions.

Thinking (20 percent):
Use of critical and creative thinking skills to solve problems, analyze relationships, develop strategies, prove identities, and justify mathematical reasoning.

Communication (20 percent):
Clarity and precision in mathematical explanations, notation, diagrams, graphs, representations, and communication of reasoning.

Application (30 percent):
Application of mathematical knowledge and skills to real-world contexts, modelling situations, rates of change, function operations, and interdisciplinary problem solving.

e. Final Evaluation (30 percent)

The final evaluation consists of:

1. Culminating Performance Task (10–15 percent)

A multi-step mathematical investigation or applied modelling project. Examples include:

- Modelling real-world phenomena using functions;
- Analyzing financial growth using sequences and exponential models;
- Designing and interpreting sinusoidal models for periodic behaviour;
- Investigating transformations and applications of functions using technology.

2. Final Examination (15–20 percent)

A proctored examination assessing conceptual understanding, algebraic fluency, mathematical reasoning, problem-solving skills, and application across all course strands.

3. Summary of Achievement Levels and Percentage Grade Ranges

Student achievement in secondary school courses is evaluated according to the Ontario Ministry of Education's four-level achievement scale. Each percentage grade corresponds to a level of achievement, as outlined below.

Percentage Grade Range	Achievement Level	Summary Description
80–100%	Level 4	Demonstrates a very high to outstanding level of achievement. Performance exceeds the provincial standard.

70–79%	Level 3	Demonstrates a high level of achievement. Performance meets the provincial standard. (This is the provincial expectation for student performance.)
60–69%	Level 2	Demonstrates a moderate level of achievement. Performance is below but approaching the provincial standard.
50–59%	Level 1	Demonstrates a passable level of achievement. Performance is significantly below the provincial standard. Further improvement is required for continued success.
Below 50%	Level 0	Insufficient achievement. Curriculum expectations have not been met. A credit will not be granted.

Academic Honesty and Integrity Policy

Inukshuk Academy is committed to fostering a learning environment based on honesty, responsibility, respect, and academic integrity. Students are expected to submit original work and properly acknowledge all sources of information, ideas, images, data, and wording used in assignments and investigations. Academic honesty is essential to maintaining fairness and trust in the evaluation process.

All submitted work must represent the student's own understanding and effort. Sources must be documented appropriately using MLA or APA citation style, as directed by the teacher. The use of generative artificial intelligence tools (e.g., ChatGPT) or other forms of external assistance must be disclosed and approved by the teacher. Undisclosed or inappropriate use of such tools may be treated as academic dishonesty.

Students may be asked to provide process work, rough drafts, notes, calculations, or planning documents to verify authorship and learning progress. If academic dishonesty occurs, the student and Principal may be notified, and consequences will be applied in accordance with school policy. Repeated incidents may result in escalating consequences.

1. Improper Citation

Improper citation occurs when a student fails to use appropriate citation formatting when referencing sources.

Grade Level	First Instance	Subsequent Instances
Grades 9–10	Warning and opportunity to resubmit	Warning and opportunity to resubmit
Grades 11–12	Warning and opportunity to resubmit	Opportunity to resubmit, maximum grade of 75%

2. Unaccredited Paraphrasing

Unaccredited paraphrasing occurs when a student rewrites another person's ideas without giving proper credit.

Grade Level	First Instance	Subsequent Instances
Grades 9–10	Warning and resubmission allowed	Resubmission allowed, maximum grade of 75%
Grades 11–12	Resubmission allowed, maximum grade of 75%	Resubmission allowed, maximum grade of 50%

3. Unaccredited Verbatim Copying

This occurs when a student copies sentences or passages directly from a source without proper citation.

Grade Level	First Instance	Subsequent Instances
Grades 9–10	Resubmission allowed, maximum grade of 75%	Resubmission allowed, maximum grade of 50%
Grades 11–12	Resubmission allowed, maximum grade of 50%	Grade of zero. No resubmission permitted

4. Full Plagiarism

Full plagiarism occurs when a student submits work entirely created by another person or source.

Grade Level	First Instance	Subsequent Instances
Grades 9–10	Grade of zero. No resubmission	Grade of zero. No resubmission
Grades 11–12	Grade of zero. No resubmission	Grade of zero and possible removal from the course

5. Missed Work

Students are responsible for reviewing lesson materials, completing missed assignments, and meeting established deadlines. Students are expected to remain informed about course expectations, due dates, and assessment requirements.

6. Homework Expectations

Students are expected to:

- Complete homework assignments, readings, and practice problems on time;
- Prepare for scheduled lessons, laboratories, discussions, and assessments;
- Participate actively and consistently in course activities and investigations.

If homework is repeatedly incomplete, the teacher may:

- Schedule a meeting with the student;
- Contact parents or guardians;
- Consult with school administration to support student success and progress.

Inclusion, accommodations & ELL supports

Inukshuk Academy is a small private school with limited accommodation resources. The school is committed to supporting students' access to learning within the scope of available staffing, programming, facilities, and course delivery structures.

Where possible, reasonable accommodations may be provided for students with documented learning needs, including limited support for Individual Education Plans (IEPs). Accommodations are considered on a case-by-case basis in consultation with the student, parents/guardians, teacher, and school administration.

Possible supports may include:

- Clear written instructions and success criteria;
- Guided notes, visual examples, and scaffolded practice;
- Reasonable extended time where appropriate;
- Preferential seating or reduced-distraction work space when available;
- Use of approved assistive technology;
- Check-ins with the teacher to support organization and progress;
- ELL support through simplified instructions, vocabulary support, and additional clarification.

Inukshuk Academy may not be able to provide specialized programming, intensive one-on-one support, modified curriculum expectations, or accommodations requiring resources beyond the school's capacity. Students requiring significant accommodations or specialized services may need additional external support.

Instructional Philosophy

Teaching and learning in MHF4U emphasize:

- Inquiry-based learning through investigations, modelling tasks, and problem-solving activities;
- Conceptual understanding supported by real-world and theoretical applications of advanced functions;
- Mathematical modelling of relationships, patterns, rates of change, and function behaviour;
- Multiple representations of functions, including equations, graphs, tables, diagrams, transformations, and written explanations;
- Collaboration through group problem solving, discussions, mathematical investigations, and peer review of solutions;
- Technology integration in graphing, modelling, data analysis, and visualizing polynomial, rational, exponential, logarithmic, and trigonometric functions.

This approach ensures students engage deeply with advanced mathematical concepts while developing transferable problem-solving, reasoning, communication, and STEM skills.

Course Learning Resources

Students will engage with a variety of mathematics learning materials, both digital and print, selected to support inquiry, modelling, problem solving, and conceptual development.

a. Primary Resources

- Teacher-created notes, examples, lessons, and practice sets;
- Advanced function investigations, modelling tasks, and problem-solving activities;
- Graphing tools and mathematical visualization resources;
- Scientific calculators and mathematics reference materials;
- *Ontario Curriculum, Grades 11–12:* Mathematics *(2008)* [2]

b. Digital Tools and Platforms

- Desmos or GeoGebra for graphing, transformations, and modelling;
- Spreadsheet software for data organization, pattern analysis, and modelling;
- Online graphing calculators and function exploration tools;
- Learning management system resources, videos, and interactive activities;
- Digital assessment, collaboration, and presentation tools.

c. Supplementary Resources

- Mathematics problem banks and practice exams;
- Real-world case studies involving polynomial, rational, exponential, logarithmic, trigonometric, and composite functions;
- Articles, videos, and multimedia related to mathematical applications;
- Review materials for algebra, functions, transformations, trigonometry, rates of change, and advanced mathematical modelling.

References

[1] Ministry of Education, "Growing Sucess," 2010. [Online]. Available: https://www.edu.gov.on.ca/eng/policyfunding/growsuccess.pdf.

[2] Ministry of Education, "Course Descriptions and Prerequisites," August 2024. [Online]. Available: https://www.dcp.edu.gov.on.ca/en/course-descriptions-and-prerequisites.